Free-Falling Autorotating Plate — A Coupled Fluid and Flight Mechanics Problem

C.R. Gallaway* and W.L. Hankey†
Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio

A computational method coupling the three-degrees-of-freedom flight mechanics equations and the two-dimensional, time-dependent Navier-Stokes equations was developed which could be used to predict the autorotating characteristics of a free-falling, two-dimensional flat plate. The two-dimensional incompressible Navier-Stokes equations were cast in a body-fixed coordinate system. The corresponding velocities were cast in an inertial reference system. The Navier-Stokes equations were represented by implicit finite differences and solved using a successive-overrelaxation iteration technique. The resulting aerodynamic coefficients were entered into the three-degrees-of-freedom flight mechanics equations. The system of ordinary differential equations describing the flight mechanics was solved using an Adams-Bashforth explicit method to predict the movement of the plate. New values of the boundary conditions for the Navier-Stokes equations solver were obtained from this movement of the plate. The process was repeated to advance the solution in time. The flight path of a free-falling autorotating plate was predicted using the computational procedure and the validity of the overall approach demonstrated by comparison with experiment.

Nomenclature

D = aerodynamic drag
I_y = mass moment of inertia
L = aerodynamic lift
M = aerodynamic moment about center of gravity
m = mass
p = static pressure
Re = freestream Reynolds number
u,v = flowfield velocities in inertial axes
V_{cg} = velocity of center of gravity of plate
x,y = coordinates in the rotating axes
γ = glide path angle
θ = Euler pitch angle

Subscripts

b = location on the body surface
o = location of origin of rotating axis system in inertial plane
x = differentiation with respect to x
y = differentiation with respect to y
∞ = ambient condition

Superscripts

$(\tilde{\ })$ = transformed nondimensional variable
$(\dot{\ })$ = differentiation with respect to time

I. Introduction

MANY aircraft accidents each year are attributed to spin resulting from aircraft departure during stall. Many such accidents may be prevented if the spin characteristics of the aircraft are better understood. However, predicting the characteristics of a spinning aircraft is very involved. Conventional wind tunnel testing of aircraft models does not accurately simulate spin. Experience has indicated that aircraft spin can be investigated safely and at a comparatively moderate cost by testing small dynamic models in a vertical wind tunnel.[1] However, because of the many variables in a spin, interpretation of spin tunnel results for application to a corresponding airplane is difficult and may show only general trends. Ultimately, it is necessary to turn to full-scale flight testing, which is extremely expensive and will become more expensive.

One is inclined to turn to numerical techniques to analyze aircraft spin because per unit computing costs are decreasing rather than increasing. Unfortunately, to investigate aircraft spin mathematically, a six-degree-of-freedom trajectory program must be coupled with a three-dimensional time-dependent Navier-Stokes program. The large computer memory requirements and long run times make this task impractical on today's computers.[2] While one awaits larger computers to calculate aircraft spin precisely, one can search for a simpler problem that can be accurately solved on present computers. However, developing and demonstrating a procedure to solve a simpler problem that incorporates the essential elements of a spinning aircraft will be a step toward solving the complete aircraft spin problem.

The free-falling, autorotating flat plate incorporates the essential elements (unsteady aerodynamics, flight mechanics, and autorotation) of a spinning aircraft. When a rectangular piece of paper (e.g., a computer card) is dropped, it falls obliquely and not vertically, as might be expected.[3] Presently, the flowfield around the free-falling, autorotating flat plate can be modeled in two dimensions and the flight characteristics numerically predicted using present computers.

The purpose of the present research is to develop and apply a method for solving the three-degree-of-freedom flight mechanic equations that can be used to predict the flight path of a free-falling, autorotating, two-dimensional object. The three degrees of freedom will be vertical and horizontal translation and rotation (x, y, ϕ). Simultaneously the unsteady, nonlinear aerodynamic forcing functions (lift, drag, and moment) in the flight mechanic equations will be calculated by solving the two-dimensional incompressible Navier-Stokes equations. These two systems of equations can then be coupled to solve the complete problem and permit comparison with experimental data.

Presented as Paper 84-2080 at the AIAA Atmospheric Flight Mechanics Conference, Seattle, WA, Aug. 21-23, 1984; received Sept. 24, 1984; revision received June 5, 1985.

*Aerospace Engineer, Defense Intelligence Agency.
†Senior Scientist.

II. Numerical Approach

A numerical technique was developed which simultaneously combined a two-dimensional time-dependent Navier-Stokes aerodynamic prediction program with a three-degree-of-freedom trajectory program.[4] The Navier-Stokes equations were solved at a particular instant in time. The aerodynamic forces predicted by the Navier-Stokes equations were input into the flight mechanics equations, which were then solved to obtain the movement of the plate. The movement of the plate was then used to obtain a new set of boundary conditions for the Navier-Stokes equations, and the cycle was repeated.

Navier-Stokes Equation Solver

The unsteady, nonlinear aerodynamic characteristics of a rotating flat plate were approximated by a finite difference solution of the two-dimensional incompressible Navier-Stokes equations. The flowfield around the flat plate was computed using a finite difference grid, shown in Fig. 1, which clustered points near the surface. The grid in the physical plane was transformed to produce evenly spaced points in the computational plane, which simplified the solution of the finite difference equations.

The Navier-Stokes equations were formulated with primitive variables (u, v, p). In the past, two traditional methods have been used to express the coordinate system for rotating bodies, i.e., inertial or rotating axes. However, both systems were found to be unsatisfactory for this problem. Inertial coordinates required the generation of a new grid for each time step, while the rotating system produced excessively high transformed velocities at the outer boundary. To alleviate this problem, a "mixed" coordinate system was devised in which the spatial coordinates were expressed in a rotating body-fixed system, while the velocity components were described in an interial framework. The mixed coordinate system is shown in Fig. 2. In this manner numerical difficulties were eliminated.

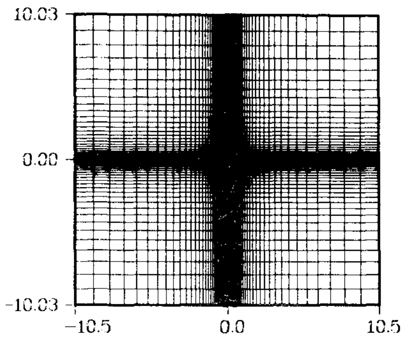

Fig. 1 Physical domain.

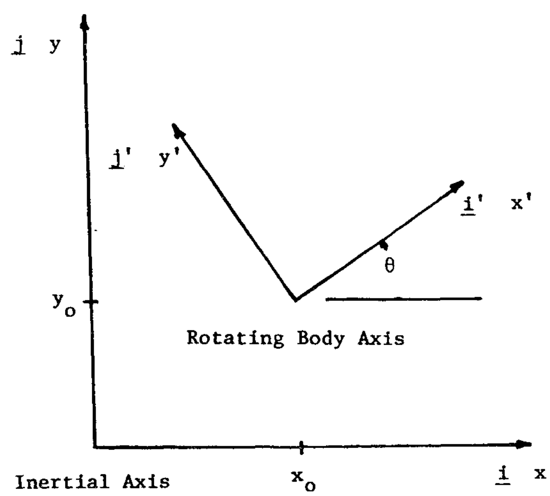

Fig. 2 "Mixed" coordinate system.

The time-dependent, two-dimensional incompressible Navier-Stokes equations cast in the mixed system are given by

$$\tilde{u}_t + [\tilde{u} + y\dot{\theta} - (\dot{x}_o\cos\theta + \dot{y}_o\sin\theta)]\tilde{u}_x + [\tilde{v} - x\dot{\theta} - (-\dot{x}_o\sin\theta + \dot{y}_o\cos\theta)]\tilde{u}_y = -p_x + [1/Re(\tilde{u}_{xx} + \tilde{u}_{yy})] + \dot{\theta}\tilde{v}$$

$$v_t + [\tilde{u} + y\dot{\theta} - (\dot{x}_o\cos\theta + \dot{y}_o\sin\theta)]\tilde{v}_x + [\tilde{v} - x\dot{\theta} - (-\dot{x}_o\sin\theta + \dot{y}_o\cos\theta)]\tilde{v}_y = -p_y + [1/Re(\tilde{v}_{xx} + \tilde{v}_{yy})] - \dot{\theta}\tilde{u}$$

$$\tilde{u}_x + \tilde{v}_y = 0$$

where $\tilde{u} = u\cos\theta + v\sin\theta$, $\tilde{v} = -u\sin\theta + v\cos\theta$ } contravariant velocity components

This set of partial differential equations was converted to a system of algebraic equations by a finite difference representation. At each nodal point of the computational grid of Fig. 1, the algebraic equations required the conservation of mass and momentum. The finite difference set of equations were solved with a second-order-in-space, first-order-in-time implicit scheme. The details of the numerical procedure are described in Refs. 4-6. The solution of these equations produced the velocity and pressure fields around the flat plate. The aerodynamic coefficients were obtained by numerical integration of the pressure distribution on the surface of the plate.

Flight Mechanics Equation Solver

The second part of the problem required the solution of the three-degree-of-freedom flight mechanics equations cast in the wind axes reference system. This was an initial value problem involving the following system of three nonlinear ordinary differential equations (two first order and one second order):

$$-m\dot{V} = D + W\sin\gamma, \quad mV\dot{\gamma} = L - W\cos\gamma, \quad I_y\ddot{\theta} = M$$

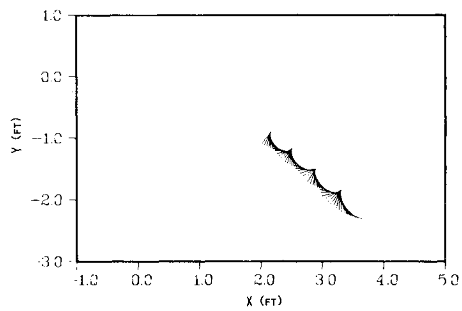

Fig. 3 Experimental plate position during free-fall autorotation.

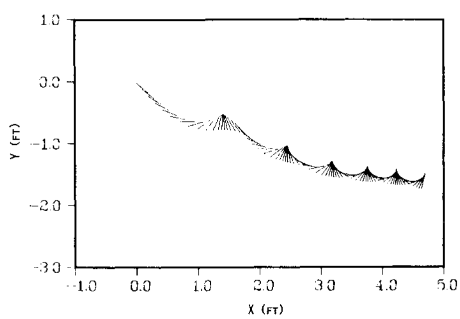

Fig. 4 Calculated plate position during free-fall autorotation, $\gamma_o = -\pi/4$, $V_o = 3.0$, $\theta_o = -0.1$.

Several methods exist for solving systems of first-order equations; therefore, the second-order equation was reduced to two first-order equations. The equations were approximated by a system of finite difference equations that were solved using an Adams-Bashforth explicit method.

Initial Conditions

Experiments[7] with freely falling plates indicated that a wide variety of arbitrary initial conditions produced the same autorotation characteristics. For this reason, a simple and convenient method was selected to initiate the numerical program.

The plate was first oriented at a fixed angle of attack and constant flight velocity, and a steady-state numerical solution of the Navier-Stokes equations was obtained. This state was then used as the initial condition for the flight mechanics phase. Upon release of the plate, the time-dependent fluid and flight mechanics equations were coupled and solved simultaneously.

Coupled Systems

The aerodynamic forces produced by the plate that were calculated from the pressure and velocity field around the plate were entered into the flight mechanics as "forcing functions" (D, L, M). The same time step used in the Navier-Stokes equations solver was used in the flight mechanics equations solver. (A systematic investigation of a suitable time step was accomplished in Ref. 4 in order to obtain the desired accuracy.) The flight mechanics equations were solved for the next time step. This solution produced a new translational velocity, glide path angle, rotation angle, and rotational velocity.

These conditions for the movement of the plate (V, γ, ϕ), were imposed as surface boundary conditions on the plate for the Navier-Stokes equation solver using the following equations:

$$u_b = V_{cg}\cos(\theta-\gamma) - y\dot{\theta} \qquad v_b = V_{cg}\sin(\theta-\gamma) + x\dot{\theta}$$

The outer boundary conditions remained as no-flow ($u=0$, $v=0$) and constant ambient pressure (p_∞). A new flowfield and the corresponding aerodynamic coefficients were calculated for the new time step. The motion of the free-falling plate was calculated by repeating the aforementioned process thousands of times to predict the motion of an autorotating plate.

III. Results

Experiment

To acquire quantitative information about an autorotating plate, the path of a free-falling rectangular card was recorded by means of high-speed photography.[7] The position of the two edges of the card were digitized for each photographic frame. An example of the motion is shown in Fig. 3 for a card with the physical characteristics listed in Table 1.

By numerically differentiating these values, the flight parameters (V_{cg}, θ, and γ) were obtained. Additional differentiations of these parameters with respect to time permitted the extraction of the aerodynamic parameters L, D, M by use of the three flight mechanics equations previously described in Sect. II.

Although the accuracy of the procedure degraded with each differentiation of the digital measurement, useful information

Table 1 Physical characteristics of the plate

Chord, in.	3
Span, in.	8
Aspect ratio	2.67
Weight, lb	7.0×10^{-3}
Moment of inertia, $lb_f s^2 ft$	1.13×10^{-6}

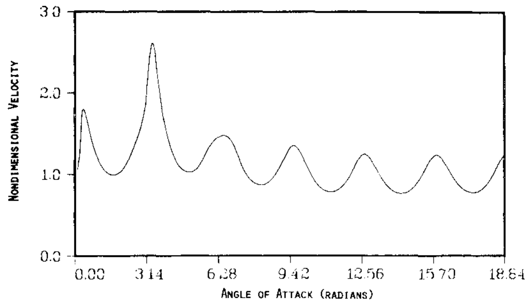

Fig. 5 Nondimensional rate of rotation of a flat plate in free-fall autorotation.

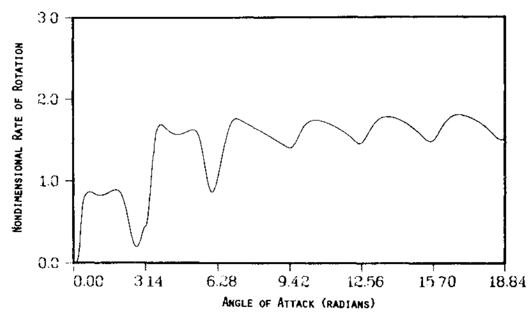

Fig. 6 Nondimensional velocity of the center of gravity of a flat plate in free-fall autorotation.

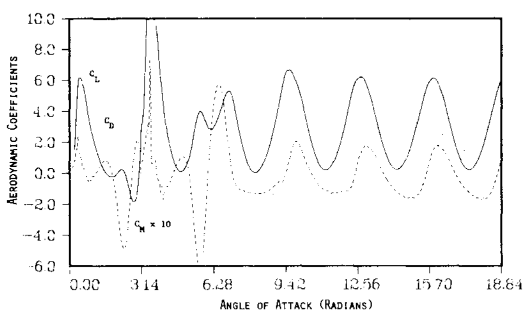

Fig. 7 Calculated aerodynamic coefficients for a flat plate in free-fall autorotation.

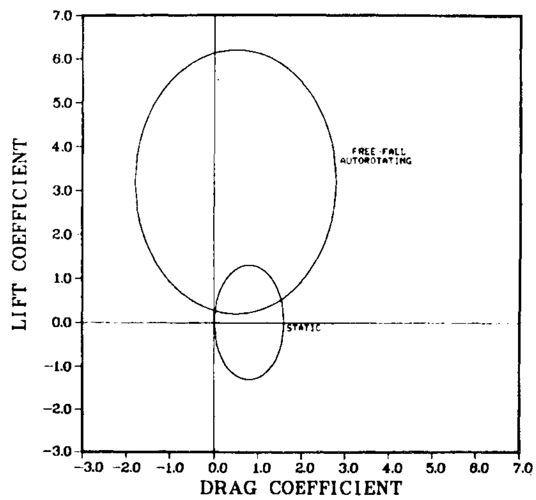

Fig. 8 Comparision of static and dynamic drag polars.

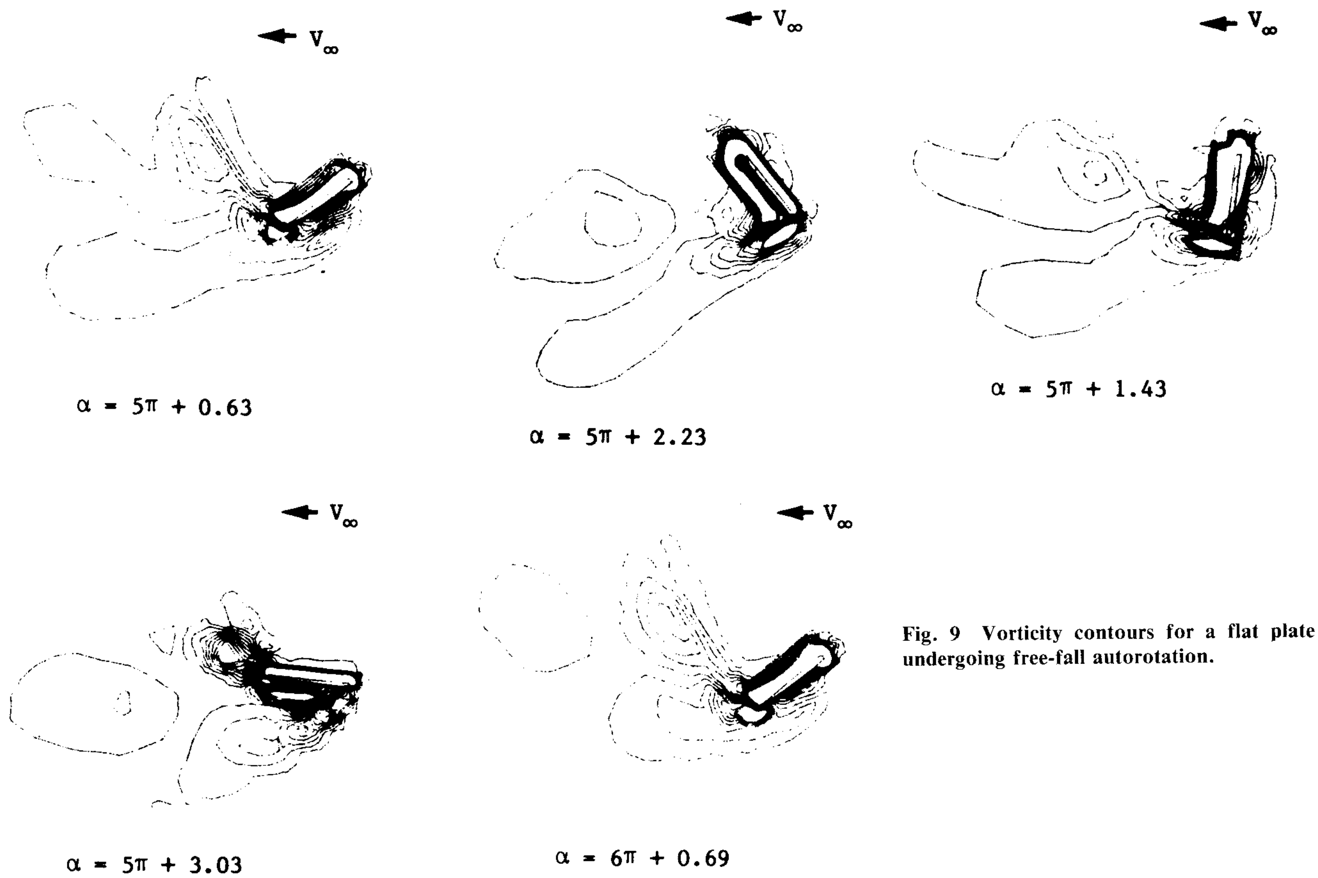

Fig. 9 Vorticity contours for a flat plate undergoing free-fall autorotation.

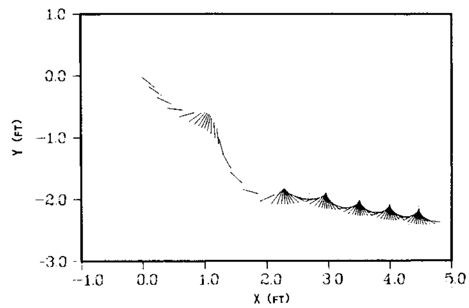

Fig. 10 Calculated flat plate position during free-fall autorotation, $\gamma_o = -\pi/4$, $V_o = 3.0$, $\theta_o = -0.1$.

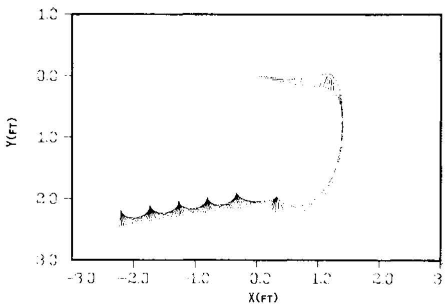

Fig. 11 Calculated flat plate position during free-fall autorotation, $\gamma_o = 0$, $V_o = 6.0$, $\theta = 0.1$.

concerning the motion was obtained. Reference 7 provides further details concerning the experiment.

Computation

The computational procedure described in Sec. II was used to compute the free fall of a plate with the physical characteristics presented in Table 1. The initial conditions for the plate were moving down and to the right with a velocity of 3.0 ft/s at a glide path angle of $-\pi/4$ rad. The plate initially at a θ angle of $-\pi/4+0.1$ rad produced an angle of attack of 0.1 rad. The plate was "released" and allowed to free-fall by simultaneously solving the coupled system of equations. The plate established autorotation within two revolutions as depicted in Fig. 4. The trace of the numerically predicted flight path began at the point where the card was released and allowed to free-fall. Note the similarity of the computed motion with the experimental data of a plate with identical physical characteristics obtained using high-speed photography[7] shown in Fig. 3.

The calculation of the free-falling plate produced an autorotation after an initial transient (Fig. 4). The plate path during autorotation produced an inverted-cycloid pattern (Fig. 4), which was also observed in the experiment (Fig. 3). During this motion the computed magnitude of the translational velocity of the center of gravity and the rotational rate were changing periodically, as illustrated in Figs. 5 and 6. A unique phase relationship between the translational and rotational velocities was required to produce this appearance of the plate rotating alternately about its edges.

The plate in the numerical simulation wa released at 3.0 ft/s and developed into autorotation with a mean velocity of approximately 3.01 ft/s. The mean velocity of the experimental plate was approximately 3.73 ft/s. The plate in the numerical simulation autorotated at a mean rotational velocity of 19.56 rad/s, while the experimental plate was found to rotate at approximately 23.4 rad/s. This agreement in the mean translational and rotational velocities was considered satisfactory at this phase of the research; however, a much larger discrepancy in the mean flight path angle was observed between the experiment and computation. Additional effort is required in the future to increase the accuracy of both the experiment and computation.

The aerodynamic coefficients encountered in the free-falling autorotation calculations were considerably larger than the coefficients obtained at static angles of attack. The calculated aerodynamic coefficients produced by the plate as a function of angles of attack are shown in Fig. 7. A comparison between the static drag polar for a flat plate and the drag polar for a free-falling, autorotating flat plate are shown in Fig. 8. The large differences between the static and dynamic aerodynamic coefficients are the primary reason that past attempts to calculate autorotation using quasisteady numerical techniques have failed. The unsteady, nonlinear aerodynamic forces are an essential ingredient in autorotation.

For the first time since initial attempts were made to explain the free-falling, autorotating flat plate in 1854 (Ref. 3), the unsteady flowfield around the plate is known. The lines of constant vorticity at several angles of attack are shown in Fig. 9. Each plot has been aligned so that the freestream velocity is always coming from the right. The vorticity contours illustrate the asymmetry caused by the rotation.

The influence of varying initial conditions on the autorotation motion was explored. The plate was released similarly to the previous case; however, the initial velocity was increased to 6.0 ft/s. The initial flight path was slightly different, as shown in Fig. 10, but the fully developed autorotation was identical to the previous case.

In another launch mode the plate was initially moving to the right with a velocity of 6.0 ft/s at a glide path angle of -0.1 rad and a fixed angle of attack of 0.1 rad. The plate was released and reversed direction as shown in Fig. 11. The plate did not have sufficient inertia to rotate past the portion of the flight, where it experienced a retarding moment. The plate stopped rotating, reversed direction, and established autorotation, falling in the opposite direction. After autorotation was established, the flight characteristics of the plate were identical to the previous free-fall autorotation cases except that the plate was moving in the opposite direction. A similar flight path was demonstrated experimentally by the author. Numerical experimentation with the different launch modes indicates that the plate will establish the same autorotation mode independently of the launch mode.

IV. Conclusions

The goal of this research was to develop a method of coupling an unsteady, nonlinear aerodynamic prediction technique with the flight mechanics equations and to demonstrate the method by predicting the motion of a free-falling, autorotating flat plate. A numerical technique that combined a two-dimensional Navier-Stokes aerodynamic prediction program with a three-degrees-of-freedom trajectory program was developed and used to predict the motion. Confirmation of the numerical approach was achieved by comparison with the experimentally observed autorotation characteristics.

The discrepancy between the experiment and computational quantitative results was greater than desired. To resolve this issue, it is recommended that better photographic resolution (in both space and time) of the rotating plate be acquired in order to improve the experimental precision and also that the computations be repeated with a large number of grid points in order to reduce the numerical spatial errors.

Developing and demonstrating a procedure to predict numerically a physical phenomenon that incorporates the essential elements of a spinning aircraft is a pathfinder. This study was the first time an unsteady, nonlinear aerodynamic prediction technique was used in solving the three-degrees-of-freedom equations of motion. The feasibility of simultaneously solving the two sets of equations through an iterative process was demonstrated. Identifying the mixed reference system for future applications to spin computations was an important contribution. This pathfinder has laid the ground work for future numerical analysis of the complete aircraft problem.

References

[1]Neilhouse, A.I. et al. "Status of Spin Research for Recent Airplane Designs," NASA TR R-57, 1960.

[2]Hankey, W.L., "Introduction to Computational Aerodynamics," AFWAL-TR-82-3031, April 1983.

[3]Maxwell, J.C., "On a Particular Case of the Descent of a Heavy Body in a Resisting Medium," Scientific Papers, Cambridge University Press, 1890, p. 115.

[4]Gallaway, C.R., "Numerical Aerodynamic Analysis of a Free Falling Autorotating Plate," Ph.D. Dissertation, Air Force Insititute of Technology, WPAFB, 1983.

[5]Hodge, J.K., Stone, A.L., and Miller, T.E., "Numerical Solution for Airfoils Near Stall in Optimized Boundary-Fitted Curvilinear Coordinates," *AIAA Journal,* Vol. 17, May 1979, p. 458.

[6]Hegna, H.A. "Numerical Prediction of Dynamic Forces on Arbitrarily Pitched Airfoils in Turbulent Flow," AIAA Paper 82-0092, 1982.

[7]Gallaway, C.R., Graham, J.E., and Hankey, W.L., "Flight Analysis of a Free-Falling Autorotating Plate," AFWAL-TM-83-177, 1983.